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Dynamics of Compressed Optical Pulse in Cubic-quintic Media

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ABSTRACT
We consider optical pulse propagation in cubic-quintic nonlinear media and analyze the dynamics of pulse compression within the framework of variational approach. Based on a potential model we find effective pulse width for stable propagation. We see that a pulse propagating in cubic media gets compressed due to quintic nonlinearity. The value of width of a compressed pulse decreases with the increase of power. However, its minimum value is limited by the growing instability above the critical power. We explicitly examine that the compressed pulse is dynamically stable due to the competition between cubic and quintic nonlinearities.

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1. Introduction

Optical soliton plays the promising role of a signal carrier due to its capability of propagation over a long distance without attenuation. In the nonlinear optical communication systems, the pulse propagation through waveguide (WG) is usually governed by the Kerr-type (cubic) nonlinearity. However, as one increases the power of the incident signal, the scenario gets changed [1]. In this case, the effect of higher-order (non-Kerr) nonlinearities comes into play and affects the feature as well as stability of the solitonic pulse [2, 3].

One of the important effects that arises due to higher-order nonlinearity is the pulse compression (PC). A chirped solitary pulse propagating in cubic-quintic media gets compressed both in normal and anomalous dispersion regimes. Eventually, this nonlinear effect generates ultra-short pulse from a broader signal [4–7]. A linear effect can also cause compression of chirped pulse propagating through grating pairs after passing the dissipative delay line [8, 9]. Besides the above methods, one can generate a compressed pulse in air, silica glasses and liquids by the technique of filamentation and plasma generation in the high-intensity region [10, 11]. In the filamentation process one of the key parameters is the critical power (CP). If power of the input signal is

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taken far above a few tens of the CP then multiple filaments can generate due to modulational instability [12–14].

The pulse propagation in the presence of higher-order nonlinearity suffers from critical collapse due to competition between nonlinearities of different orders [15]. Investigations in the presence of focusing quintic nonlinearity show that collapse of solitary wave can be suppressed in (2 + 1) dimension either by introducing nonlocal nonlinearity [16] or by spatially modulating nonlinear terms [17–20].

The multidimensional optical pulse which is unstable in a medium with focusing cubic nonlinearity can be stable in a cubic-quintic nonlinear medium due to the competition between focusing cubic nonlinearity and defocusing quintic nonlinearity [21, 22]. Recently, stability of 1D soliton in saturable focusing quintic nonlinear media has been studied and shown that it is possible to achieve broader stability region in the presence of nonlinear lattice [23].

Experimental realization of cubic-quintic nonlinearity in some material like CdS_xSe_{1-x} doped glass and the ability to control sign and strength of nonlinearity by adjusting doping in a doubly doped fiber make the system physically more interesting [24, 25]. Recently, several studies are envisaged in the search of different types of solutions, namely, dark, bright, kink and dipolar solitons in cubic-quintic nonlinear media with dispersion and gain/loss terms [20, 26]. In addition, attempts are also made to understand the static and dynamical properties of such solutions in nonlinear optical systems [27–33].

In this work, we address how the competition between cubic and quintic nonlinearities leads to the generation of short pulse and, interplays with the group velocity dispersion while propagating through optical fiber. More specifically, we model the system by cubic-quintic nonlinear Schrödinger equation and systematically investigate dynamical and physical changes of propagating pulse. Within the framework of Ritz optimization procedure (ROP), we construct an effective potential for the pulse [34, 35]. It is seen that the effective potential becomes minimum for a certain pulse width, the value of which is smaller than that in the absence of quintic nonlinearity. Thus quintic nonlinearity causes pulse compression. This compression is found to increase with the increase of power (E_0). However, beyond the limiting value of E_0 the pulse becomes linearly unstable [36].

In section II, we derive the model equation for the pulse propagation in a cubic-quintic nonlinear medium from the Maxwell's equation using slowly varying envelop approximation (SVEA). In Section III, we present Lagrangian based variational approach and find equations of different parameters of the pulse. In section IV, we present an analysis based on the effective potential and discuss the interplay of different terms in the creation of stable solitonic wave. In this context, we note that this approach is found very effective in describing the dynamics of soliton in other nonlinear systems, namely, the matter-wave solitons in Bose-Einstein condensates [37–39]. In section IV, we present linear stability analysis based on Vakhitov-Kolokolov criterion and dynamical stability based on numerical simulation. We conclude by noting some results in section V.

2. Theoretical model

Generally, when high-intensity optical pulses are considered, it is necessary to take into account higher order nonlinearities arising from an expression of the refractive index in powers of intensity I of the light pulse. The wave equation for high-intensity light pulse propagation in an isotropic single-mode optical fiber with a circular cross-section and fiber axis z can be written as

$$ {}^{2}\varepsilon - \frac{n^2(\omega)}{c^2}\frac{\partial^2 \varepsilon}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \tag{1}$$

where c is the velocity of light in vacuum. For the high-intensity light pulse propagation, the nonlinear polarization is given by

$$ P_{NL} \approx P_{NL}^{(3)} + P_{NL}^{(5)} = \frac{3}{4}\epsilon_0 \chi^{(3)} |\varepsilon_0|^2 \varepsilon + \frac{5}{4}\epsilon_0 \chi^{(5)} |\varepsilon_0|^4 \varepsilon \tag{2}$$

where ϵ_0 is the permittivity of vacuum and $\chi^{(k)}$ $(k = 1, 2, 3 \ldots)$ is the kth susceptibility. Understandably, the third ($\chi^{(3)}$) and fifth-order ($\chi^{(5)}$) susceptibilities are responsible for nonlinear effects in fiber. All the even-order nonlinear susceptibility coefficients are equal to zero in optical fiber since the fiber is made out of symmetric molecule, like silica, for which $\chi^{(2)}$ vanishes. However, a medium which lacks inversion symmetry at the molecular level has non-zero value of $\chi^{(2)}$. In writing Eq.(2) we have used $\varepsilon = \varepsilon_0 e^{i(kz-\omega t)}/2 + c.c$, (where c.c stands for complex conjugate). The presence of P_{NL} clearly indicates that the refractive index $n_0(\omega)$ gets modified to $n(\omega)$ such that

$$ n(\omega, I) = n_0(\omega) + n_2 I + n_4 I^2 \tag{3}$$

where $I = \frac{1}{2}\epsilon_0 n_0 c |\varepsilon_0|^2$, $n_2 = \frac{3}{4}\frac{\chi^3}{\epsilon_0 c n_0^2(\omega)}$, and $n_4 = \frac{5}{4}\frac{\chi^5}{\epsilon_0 c^2 n_0^3(\omega)}$. Understandably, $n_0(\omega)$ represents refractive index (r.i.) of the linear part while n_2, n_4 are the r.i. of non-linear parts.

We consider the following separation of variable

$$ \varepsilon(r, \phi, z, \omega) = \frac{1}{2}\left(F(r, \phi) A(z, \omega) e^{i(\beta_0 z - \omega t)} + c.c \right) \tag{4}$$

for the field distribution $F(r, \phi)$ and the envelope $A(z, \omega)$, and assume that the non-linearity has no influence on the transversal component of the field. Considering the SVEA, $|\frac{\partial^2 A}{\partial z^2}| \ll \frac{\partial A}{\partial z}$ we write the envelop equation

$$\frac{\partial A}{\partial z} = i\left[\frac{3}{8}\frac{\chi^3\omega}{cn}|FA|^2A + \frac{5}{16}\frac{\chi^5\omega}{cn}|FA|^4A + (\beta^2 - \beta_0^2)A\right]. \tag{5}$$

Here we approximate $(\beta^2 - \beta_0^2)$ by $2\beta_0(\beta - \beta_0)$. We introduce a normalized amplitude $\tilde{\psi}(z,\omega)$ such that $\tilde{\psi}(z,\omega) = KA(z,\omega)$ with $K^2 = \pi\varepsilon_0 n_0 c\int|F(r)|^2 rdr$. This gives

$$\frac{\partial\tilde{\psi}}{\partial z} = i\left[(\beta - \beta_0)\tilde{\psi} + \gamma|\tilde{\psi}|^2\tilde{\psi} + \sigma|\tilde{\psi}|^4\tilde{\psi}\right] \tag{6}$$

where $\tilde{\psi}$ is the Fourier transform of ψ. Using Taylor series expansion of β and retaining only terms upto second order in $(\omega - \omega_0)$ we get

$$\frac{\partial\tilde{\psi}}{\partial z} = i\tilde{\psi}[(\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2\beta_2] + i\gamma|\tilde{\psi}|^2\tilde{\psi} + i\sigma|\tilde{\psi}|^4\tilde{\psi}. \tag{7}$$

In the time domain Eq.(7) can be written as

$$i(\frac{\partial}{\partial z} + \frac{1}{v_g}\frac{\partial}{\partial t})\psi - \frac{\beta_2}{2}\frac{\partial^2\psi}{\partial t^2} + \gamma|\psi|^2\psi + \sigma|\psi|^4\psi = 0 \tag{8}$$

We consider a transformation $\tau = t - \frac{z}{v_g}$ that changes (z,t) to (z,τ). The new frame of reference moving with the group velocity (v_g) is called the retarded frame. In this reference system the equation of pulse envelop obtained from Eq.(8) is written as

$$i\frac{\partial\psi}{\partial z} = \alpha\frac{\partial^2\psi}{\partial\tau^2} + \kappa|\psi|^2\psi + \beta|\psi|^4\psi \tag{9}$$

where $\alpha = \frac{\beta_2}{2}$, $\kappa = -\gamma$ and $\beta = -\sigma$. Eq.(9) is the so-called *Cubic-Quintic Nonlinear Schrödinger Equation* which can describe a short pulse propagation in optical fiber. We introduce $\psi(z,\tau) = \phi(\tau)e^{i\omega z}$ which gives

$$\alpha\frac{d^2\phi}{d\tau^2} + \kappa\phi^3 + \beta\phi^5 = -\omega\phi \tag{10}$$

A particular solution of Eq. (9) found by solving Eq. (10) for $\alpha = -1$, $\kappa = -4$ and $\beta = -3\rho$ with $\rho = \pm 1$ is given by [40]

$$\psi(\tau,z) = \frac{\sqrt{\omega}}{\left(1 + \sqrt{1 + \omega\rho}\cosh(\sqrt{2}\omega\tau)\right)^{1/2}}e^{i\omega z}. \tag{11}$$

It is a single hump bright soliton solution (right panel of Figure 1). Interestingly, Eq. (10) gives different solution for a set of parameters other than that used to construct Eq.(11). In this case, the pulse shape may change in a complicated manner which may result pulse compression/decomposition or

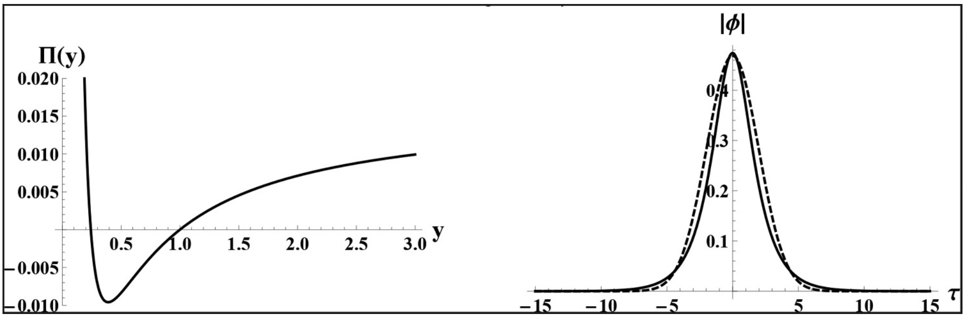

Figure 1. Left panel: Effective potential for the parameters corresponding to the solution in Eq, (11). Right Panel: Exact analytic solution (solid line) and variational solution(dashed line) of cubic-quintic nonlinear system with $\alpha = -1$, $\kappa = -4$ and $\beta = -3\rho$ with $\rho = 1$ (focusing nonlinearity). Here we have taken normalized pulse width $a_0 = 5$ and norm $E_0 = 0.437$. Width and amplitude of the stable pulse are calculated from $a(z) = a_0 y_m$ and $A(z) = \sqrt{E_0/a(z)}$ where y_m stands for the value of y corresponding to the minimum value of $\Pi(y)$.

pulse splitting. In the next section, we will discuss pulse compression dynamics based on approximation methods.

3. Variational formulation

We have seen that the stationary solution of Eq.(9) supports a single hump bright soliton. With a view to understand the effect of quintic non-linearity on the optical pulse propagating in a cubic medium we consider variational approach. We begin with the inverse variational method and write Lagrangian density $\mathcal{L}$ for Eq. (11) as

$$\mathcal{L} = i\left(\psi^*\psi_z - \psi\psi_z^*\right) + 2\alpha\left|\frac{\partial\psi}{\partial\tau}\right|^2 - \kappa|\psi|^4 - \frac{2}{3}\beta|\psi|^6 \tag{12}$$

and adopt

$$\psi(z,\tau) = A(z)e^{-\frac{\tau^2}{2a^2(z)}+ib(z)\tau^2} \tag{13}$$

as a trial solution for the propagating pulse. Here the complex amplitude $A(z)$, the pulse width $a(z)$, and the frequency chirp $b(z)$ are treated as the variational parameters. It is worth to mention that the assumption is reasonable since stationary counterpart of Eq, (1) can support Gaussian-shaped solution (Figure 1). We assume that the parameters of the pulse can vary with distance as it propagates in the medium.

One may systematically proceed with the variational method [34, 35] and write effective Lagrangian from Eqs. (12) and (13) as:

$$\langle \mathcal{L} \rangle = \int_{-\infty}^{+\infty} \mathcal{L} d\tau$$

$$= \sqrt{\pi}\left[ia\left(A^*\frac{dA}{dz} - A\frac{dA^*}{dz}\right) - a^3|A|^2\frac{db}{dz} + \alpha|A|^2a^3\left(\frac{1}{a^4} + 4b^2\right) - \frac{1}{\sqrt{2}}\kappa|A|^4a - \frac{2}{3\sqrt{3}}\beta|A|^6a\right]. \quad (14)$$

We consider the Ritz optimization procedure where the variational derivatives $\frac{\delta\langle\mathcal{L}\rangle}{\delta A^*}$, $\frac{\delta\langle\mathcal{L}\rangle}{\delta A}$, $\frac{\delta\langle\mathcal{L}\rangle}{\delta a}$ and $\frac{\delta\langle\mathcal{L}\rangle}{\delta b}$ are made to vanish for the trial solution in Eq. (13). This allows us to write

$$a(z)|A(z)|^2 = a_0|A_0|^2 = E_0, \quad (15)$$

$$\frac{da}{dz} = -4\alpha ab \quad (16)$$

and

$$\frac{d^2a}{dz^2} = \frac{4\alpha^2}{a^3} - \sqrt{2}\kappa\alpha\frac{E_0}{a^2} - \frac{8\beta}{3\sqrt{3}}\alpha\frac{E_0^2}{a^3}. \quad (17)$$

In Eq. (15), E_0 can be interpreted as power of the pulse which does not change while propagating. We have checked that E_0 is related to the norm $\mathcal{P} = \int|\psi(z,\tau)|^2 d\tau$ of the system by a numerical factor. Eq.(17) can be rewritten as

$$\frac{1}{2}\left[\frac{da}{dz}\right]^2 + \Pi(a) = 0. \quad (18)$$

It gives the dynamics of $a(z)$. This equation is analogous to that of a particle moving in a potential $\Pi(a)$ given by

$$\Pi(a) = \frac{2\alpha^2}{a^2} - \sqrt{2}\kappa\alpha\frac{E_0}{a} - \frac{4\beta\alpha}{3\sqrt{3}}\frac{E_0^2}{a^2} + c \quad (19)$$

where c is a constant to be determined from the initial conditions.

In terms of normalized pulse width $y(z) = \frac{a(z)}{a_0}$ it is convenient to introduce the following parameters

$$\Omega_1 = \frac{2\alpha^2}{a_0^4},\ \Omega_2 = \sqrt{2}\frac{\alpha\kappa E_0}{a_0^3},\ \Omega_3 = \frac{4\beta\alpha E_0^2}{3\sqrt{3}a_0^4} \text{ and } K = \frac{c}{a_0^2}. \quad (20)$$

This yields

$$\frac{1}{2}\left[\frac{dy}{dz}\right]^2 + \Pi(y) = 0 \quad (21)$$

with

$$\Pi(y) = \frac{\Omega_1}{y^2} - \frac{\Omega_2}{y} - \frac{\Omega_3}{y^2} - (\Omega_1 - \Omega_2 - \Omega_3). \tag{22}$$

In writing Eq. (22) we have used $y|_{z=0} = 1$ and $\frac{dy}{dz}|_{z=0} = 0$. This equation gives an effective potential for a short pulse propagation in optical fiber. We know that α stands for the group velocity dispersion (GVD) which is positive for normal dispersion and it is negative for anomalous dispersion. The constant $\Omega_1 \propto \alpha^2$ and thus it is independent of the type of dispersion. However, the nature of the terms arising from cubic (Ω_2) and quintic (Ω_3) nonlinearities depend on the sign of α. More specifically, Ω_2 and Ω_3 are negative for normal dispersion whereas they are positive if the dispersion is anomalous.

4. Analysis of $\Pi(y)$ on the formation of compressed pulse

Before going with the detailed analysis of $\Pi(y)$, it is an interesting curiosity to check the efficiency of the present model calculation in producing one of the analytic solutions given in Eq. (11). In order of this, we first calculate values of Ω_i $(i = 1, 2, 3)$ for $\kappa = -4$, $\beta = -3$ and $\alpha = -1$ and then find the effective potential for different values of pulse width (a). Clearly, the effective potential finds minimum at a certain value of a (left panel of Figure 1). The density distribution of the optical pulse corresponding to the minimum of the effective potential is displayed in the right panel of Figure 1. We see that the results obtained for stationary solution from variational (solid) and exact analytical (dashed) calculations show good agreement.

If the pulse starts to propagate, two possible effects come into play: (i) competition between cubic and quintic nonlinearities of the system and (ii) interplay between the dispersive effect and resulting nonlinear effect to achieve dynamical equilibrium. To visualize the role of different quantities systematically in achieving equilibrium condition we first consider the case $\kappa = \alpha = 0$. This condition is equivalent to the case of light signal propagating in air. Certainly, the pulse gets attenuated due to dispersive effects. In this limit ($\Omega_1 = 1.5$ and $\Omega_2 = \Omega_3 = 0$), the potential function is given by

$$\Pi(y) = -1.5 + \frac{1.5}{y^2}. \tag{23}$$

It is clear that there is no local minimum in the potential implying that the pulse width increases gradually and thus the pulse is dispersive. If the pulse is allowed to propagate in a dielectric medium, its propagation is affected by nonlinearity and GVD. In the limit of normal dispersion $\alpha > 0$, the potential for the typical set of parameters: $\Omega_1 = 1.5$, $\Omega_2 = -1$ and $\Omega_3 = -1$ is given by

$$\Pi(y) = \frac{2.5}{y^2} + \frac{1}{y} - 3.5 \tag{24}$$

Left panel of Figure 2 shows that pulse broadens due to the presence of dispersion and as a result no minimum is formed in the effective potential. Therefore, no stable pulse propagation is possible in this case.

Let us consider the case of anomalous dispersion $\alpha < 0$, where the nonlinearity is weak such that $\Omega_1 = 1.5$, $\Omega_2 = 1$ and $\Omega_3 = 0.002$. The potential function becomes

$$\Pi(y) = -0.498 + \frac{1.498}{y^2} - \frac{1}{y}. \tag{25}$$

In this case, we see that the effect of dispersion decreases and hence the pulse broadening reduces (middle panel of Figure 2). On other hand, the strong the quintic nonlinearity can take over all other terms, leading to wave collapse [41]. In this case no minimum is formed in effective potential (right panel of Figure 2). Therefore, it is quite expected that the competition between the nonlinearities can stop pulse broadening. In view of this, we take some appropriate quintic nonlinearity and plot in Figure 3 the effective potential

$$\Pi(y) = \frac{0.7}{y^2} - \frac{5.5}{y} + 4.8 \tag{26}$$

as a function of y. Clearly, the effective potential takes the form of a well with a sharp minimum at $y = y_{min}$. Thus the pulse can remain unaltered while propagating if the initial width is taken as a_{min}. In this case, there is a perfect balance between nonlinear and dispersive effects. However, if the pulse propagation is disturbed by some means then its width first increases by dispersive effect and then decreases by nonlinear effect (self-focussing/self-phase modulation). Thus width of the pulse oscillates during propagation. Comparing both the panel in Figure 3 we see that the value of a_{min} decreases in the

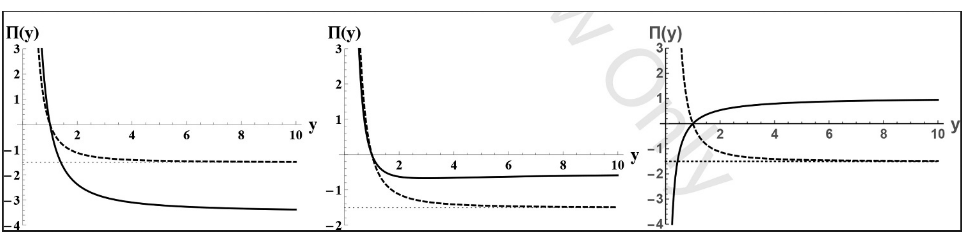

Figure 2. Left panel: Qualitative plot of the effective potential function in the normal dispersion for $\Omega_1 = 1.5$, $\Omega_2 = -1$ and $\Omega_3 = -1$. Middle Panel: Effective potential in the case of anomalous dispersion and weak quintic non-linearity for $\Omega_1 = 1.5$, $\Omega_2 = 1$ and $\Omega_3 = 0.002$. Right Panel: Effective potential in the case of anomalous dispersion and strong quintic non-linearity for $\Omega_1 = 1.5$, $\Omega_2 = 1$ and $\Omega_3 = 1.55$. In all the panels dashed line gives the potential in the linear limit($\Omega_2 = 0$, $\Omega_3 = 0$).

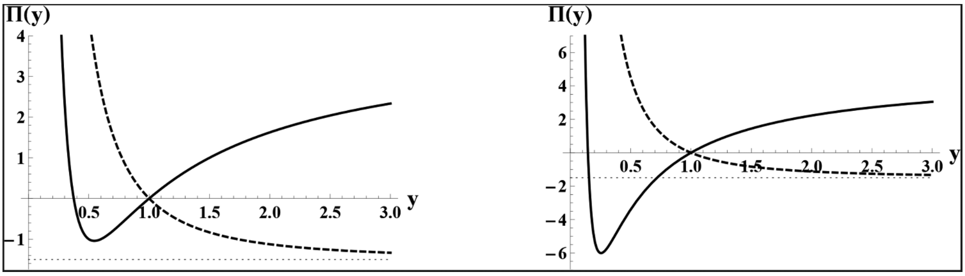

Figure 3. Qualitative plot of the potential function in the case of anomalous dispersion and strong non-linearity. Left Panel gives an effective potential without quintic non-linearity ($\Omega_1 = 1.5$, $\Omega_2 = 5.5$, $\Omega_3 = 0.0$) while right panel gives the same with quintic non-linearity ($\Omega_1 = 1.5$, $\Omega_2 = 5.5$, $\Omega_3 = 0.8$). Dashed curve gives the result for $\Omega_2 = 0$ and $\Omega_3 = 0$.

presence of quintic nonlinear term ($\Omega_3 \neq 0$). This suggests that a short pulse can be generated in cubic-quintic nonlinear media.

In Figure 4 (left panel) we display density distributions of the pulses in cubic (dashed) and cubic-quintic (solid) nonlinear media. Due to the effects of quintic nonlinearity, pulse amplitude gets augmented. However, its width decreases. If we increase power of the pulse, the pulse width will certainly be affected both in the presence and absence of quintic nonlinearity. We have checked that the pulse-compression effect due to quintic nonlinearity changes drastically with E_0. This can be understood from the expression of $\Pi(y)$ in Eq. (22). It shows that the effect of quintic term is proportional to E_0^2 while that of cubic nonlinear term is proportional to E_0. In the right panel we plot variation of pulse width with its power in cubic (dashed) and cubic-quintic media (solid). Clearly, the response of quintic nonlinearity becomes dominating with the increase of pulse power.

5. Stability property of optical pulse

We have seen that the pulse width gets affected with the increase of intensity of the propagating pulse. More specifically, the quintic nonlinearity comes into play in the presence of intense pulse and thus causes pulse compression. It is quite natural that the pulse compression should have an upper limit. To describe this fact, we envisage a linear stability analysis by the use of Vakhitov-Kolokolov (VK) criterion. The VK criterion can now be employed to check the limiting value of the pulse. It serves as a necessary condition for the linear stability of solitary waves. According to this criterion the optical soliton/pulse can be stable if $dE_0/d\omega > 0$. In principle, one can solve Eq.(10) numerically for different values of E_0 and ω to check the VK criterion [38,42]. Since our interest is to work within analytical framework, we employ variational approach and write an effective Lagrangian L from Eq.(12) using stationary

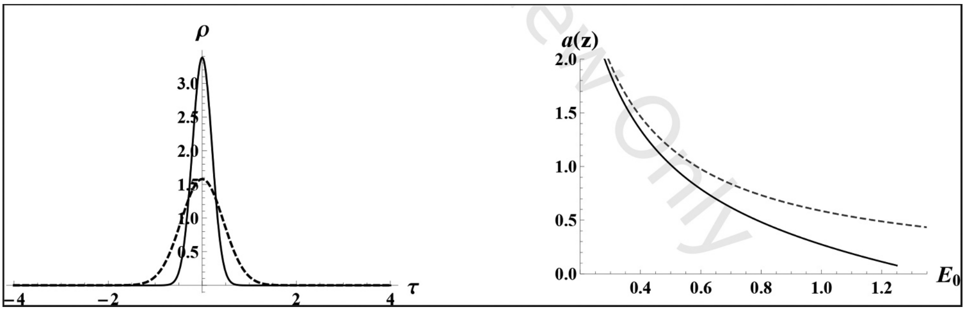

Figure 4. Effect of quintic non-linearity on the soliton solution of cubic NLS equation for values of parameters (Ω_i, $i = 1, 2, 3$) similar to that used in Figure 3. The physical parameters in Eq. (9) can be calculated from Eq. (20) to write $\alpha = -1$, $\beta = -1.38576$ and $\kappa = -4.82581$. Left panel: The profiles are calculated corresponding to the potential minimum shown in Fig. 3. We calculate actual width from $a(z) = y_{min} a_0$. Here $a_0 = 1.07462$ for the power $E_0 = 1.0$. Right panel: Variation of pulse width with the initial energy E_0. In both the panels dashed and solid curves represent the cases for $\Omega_3 = 0$ and $\Omega_3 \neq 0$ respectively.

trial solution $\phi(\tau) = A\, e^{-\tau^2/2a^2}$. From the conditions $\frac{\partial L}{\partial E_0} = 0$ and $\frac{\partial L}{\partial a} = 0$ we get

$$a(z) = \frac{2\sqrt{2}\left(9\alpha - 2\sqrt{3}{E_0}^2\beta\right)}{9\kappa E_0} \tag{27}$$

and

$$\omega = \frac{9\kappa^2 E_0^2}{4\sqrt{2}\left(-9\alpha + 2\sqrt{3}E_0^2\beta\right)^2}\left(-\frac{27\alpha}{2} + \sqrt{3}{E_0}^2\beta\right). \tag{28}$$

Eq.(27) implies that the pulse width $a(z)$ gets changed with the variation of E_0 for given values of other parameters. In Figure 5 we plot ω (left panel) and $dE_0/d\omega$ (right panel) as function of pulse power (E_0). We see that the pulse with norm, say, $E_0(say,\ E_{0c}) > 1$ (approx.) is unstable since it fails to satisfy the VK criterion. Understandably, upper limit of E_0 for a stable solitonic pulse is determined by the values of α, κ and β. The dashed curve in Figure 4 clearly indicates that the upper limit of E_0 for the existence of stable pulse increases for a relatively weaker quintic nonlinearity. Note that the dashed curve in Figure 5 corresponds to the density profile in cubic-quintic nonlinear media shown in Figure 4. Thus the compressed pulse is linearly stable. In this context we would like to mention that the case $E_0 \gg E_{0c}$ induces modulational instability and generates multiple filaments [12–14].

The potential model clearly demonstrates that the interplay between cubic-quintic nonlinearity and group velocity dispersion (GVD) is crucially important for the formation of short pulse in an optical medium. It is now an interesting curiosity to examine the dynamical interplay of the different

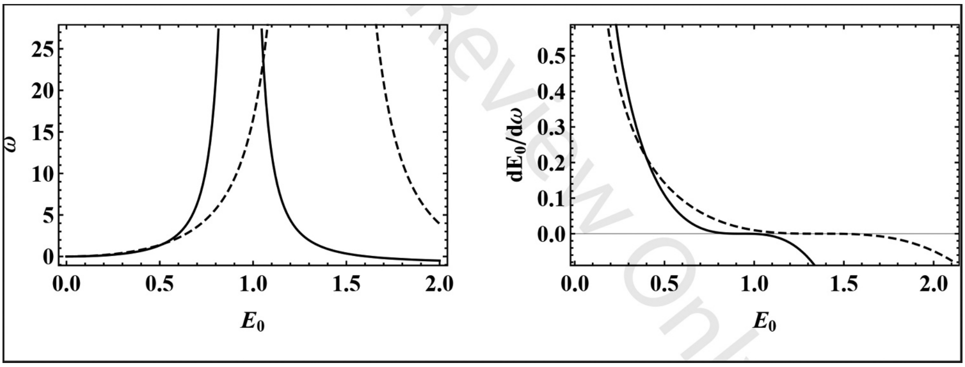

Figure 5. Left panel: Variation of ω with the initial power E_0 for a stationary solution. Right panel: Changes of $dE_0/d\omega$ with the initial power E_0. The solid curve is calculated for $a = -1$, $\kappa = -4$, $\beta = -3$ while dashed curve is calculated for $a = -1$, $\kappa = -4.8258$ and $\beta = -1.38576$. The solid and dashed curves give results in the presence of cubic-quintic nonlinearity for the density density profiles shown in Figs. 1 and 4 respectively.

higher-order nonlinearities. In view of this, we study time evolution of the pulse based on a purely numerical routine. More specifically, we make use of the split-step Fourier method to solve Eq.(10) with the initial condition $\psi(\tau, 0) = A\, exp[-\tau^2/(2a^2)]$. The results are displayed in Figure 6. We see from the left panel that the density profile (dashed line) of the compressed pulse in cubic-quintic nonlinear media remains unaltered (solid line) while evolving in time(z). This refers to dynamical stability of the compressed pulse.

In order to understand dynamical equilibrium due to the competition between cubic and quintic non-linearities we take values of κ and β other than those used in the left panel of Figure 6 and calculate density after a sufficiently large time ($z \geq 1000$). It may be noted that the dynamical equilibrium is maintained due to counter balance between dispersive and nonlinear effects. If it is disturbed by increasing the focussing quintic

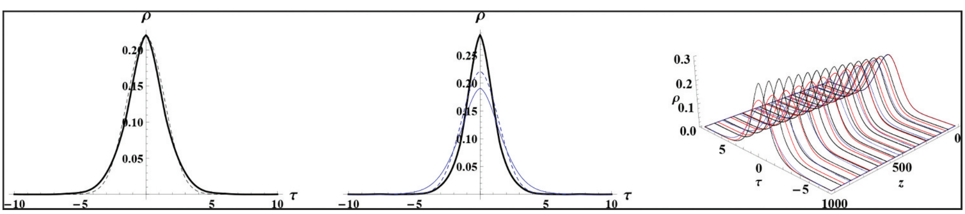

Figure 6. Left panel gives density profile calculated at $z \geq 1000$ for $a = -1$, $\kappa = -2.2834$, $\beta = -2.09880$, $E_0 = 0.437$, $a_0 = 1.980714$, $A_0 = \sqrt{E_0/a_0}$. Here dotted and dashed curves stand for the results obtained from variational and numerical calculations. Middle panel gives density profile obtained from numerical simulations: Here thick line gives density for $|\kappa| > 2.2834$ and $|\beta| = 2.09880$ while thin blue line gives the same for $|\kappa| = 2.2834$ and $|\beta| > 2.09880$. In the right panel, black and blue curves give detail of time evolution of the density profiles corresponding to the black and blue curves displayed in the middle panel while red curve represents the same for the dashed blue curve in the middle panel.

nonlinearity then a new dynamical equilibrium sets up in the system. This causes the initial pulse to get squeezed and hence gives more shorter (femto-second) pulse (black solid curve in the right panel of Figure 6). The squeezing of the pulse is, however, limited by critical collapse [20]. On the other hand, if the equilibrium is disturbed by increasing cubic nonlinearity then the pulse becomes wider and thus leading toward the pico-second regime (blue thin curve). The time evolution of a pulse shown in the right panel exhibits a detail of compression (black) or broadening (blue) of an initial pulse. We remark from the potential model that the width gets a different optimum value to establish a new dynamical equilibrium.

6. Conclusions

In general, the pulse compression process stands for a technique where a narrow pulse is produced from a wider one with a view to increase the Range of Resolution of the transmitted pulse. This technique finds potential applications in radar, sonar and echography for reducing noise to signal ratio. In optical communication, the method of compressing pulse duration starts in picosecond or femtosecond region. Here both linear and nonlinear compression techniques can be used to generate ultra-short pulse. Nonlinear compression is one of the major mechanisms in optical waveguide where one obtains soliton compression. In this paper we have derived a model equation or the so-called cubic-quintic nonlinear Schrödinger equation (CQNLSE) and described the fact that a dielectric medium with quintic non-linearity can be used to propagate an optical pulse of relatively shorter width.

We have formulated the problem within the framework of variational approach and shown that the effective potential of pulse width attains a minimum value for undistorted propagation. By the use of potential model we have demonstrated the competition between cubic and quintic non-linearities and their interplay with the group velocity dispersion in the formation of short pulse. We have envisaged linear stability analysis based on Vakhitov-Kolokolov criterion. It is seen that the pulse gets compressed gradually with the increase of its initial power/norm (E_0). However, the pulse becomes unstable if E_0 goes beyond a limiting value determined by the properties of the medium.

We have also envisaged a numerical study with a view to analyze the prediction of variational formulation and examine dynamical stability of the propagating pulse. It is seen that the pulse shape and energy remain unchanged for a typical set of parameters. If some imbalance occurs while propagating in the medium then the pulse adjusts itself according to the properties of the medium and thus maintains dynamical stability. For example, if the quintic nonlinearity increases from its equilibrium value (a value for stable propagation), we see that the pulse compression increases. In oppose to this, the pulse compression reduces if the cubic nonlinearity increases from its equilibrium. We have remarked that the squeezing effect is, however, limited by the critical collapse.

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